第2卷　第2期　　航空动力学报　　Vol.2 No.2
1987年4月　　Journal of Aerospace Power　　April 1987

轮盘振动特性计算

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摘　　要

本文提出了一种计算轮盘振动特性的数值解法。采用薄板和厚板两种理论，将问题化成一阶微分方程组进行数值计算。编制了计算机程序，得出了令人满意的结果。由于没有涉及到复杂的数学问题，计算机程序大为简化，计算时间少。

在航空涡轮发动机设计中，予测压气机和涡轮盘的动力特性具有很大的实际意义。实际涡轮盘具有任意的径向厚度变化，用纯解析方法很难分析这类结构。用有限元素法能有效地解决轮盘振动分析，但是，有限元素法有编制程序复杂，要求大的计算机存贮，计算费用高等缺陷。本文提出的方法是：从板理论的基本方程出发，导出等价的一阶微分方程组，用微分方程组数值解法求解，因此编制程序简单，省机时，易为工程技术人员掌握。对设计阶段多方案比较以及优化设计效果更明显。

一、基于薄板理论的轮盘振动数值计算

根据薄板理论，在圆柱坐标系$r\theta z$中，内力与位移之间的关系为

$$
\begin{aligned}
M_r &= -D[\partial^2 w/\partial r^2 + \mu(\partial w/r\partial r + \partial^2 w/r^2\partial\theta)] \\
M_\theta &= -D[\mu\partial^2 w/\partial r^2 + \partial w/r\partial r + \partial^2 w/r^2\partial\theta^2] \\
M_{r\theta} &= -D(1-\mu)(\partial^2 w/r\partial r\partial\theta - \partial w/r^2\partial\theta)
\end{aligned}
\tag{1}
$$

式中：M_r，M_θ为弯矩，$M_{r\theta}$为扭矩，w为板中面挠度。μ——波桑比，$D=Eh^3/12(1-\mu^2)$——抗弯刚度，E——弹性模数，h(r)——板厚度。

对于旋转盘，由于离心力、温度的作用，盘内存在平面应力。如果考虑轴对称应力状态，由离心力和温度引起的应力$\sigma_r{}^0$和$\sigma_\theta{}^0$，可由强度计算得到。用单位长度上的等效力代替，则

$$
N_r = \sigma_r{}^0 h; \qquad N_\theta = \sigma_\theta{}^0 h \tag{2}
$$

圆盘微元体平衡方程为[1]

$$
\begin{aligned}
&\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial\theta} + \frac{M_r - M_\theta}{r} - Q_r = 0 \\
&\frac{\partial M_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial\theta} + \frac{2M_{r\theta}}{r} - Q_\theta = 0 \\
&\frac{\partial(Q_r r)}{\partial r} + \frac{\partial Q_\theta}{\partial\theta} + \frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}N_r\right) + \frac{N_\theta}{r}\frac{\partial^2 w}{\partial\theta^2} - \rho h r\frac{\partial^2 w}{\partial t^2} = 0
\end{aligned}
\tag{3}
$$

式中，Q_r，Q_θ为板的横向剪力，ρ为材料密度。

本文于1986年5月收到。

轮盘的边界条件为

刚性固定边　　$W=\partial w/\partial r=0$　　(4.a)

简支边　　$W=M_r=0$　　(4.b)

自由边　　$M_r=P_r=(Q_r r+\partial M_{r\theta}/\partial\theta)=0$　　(4.c)

轮盘振动具有节圆和节径型式，位移变化具有正对称性性质。把θ函数用富里叶级数来表达，则对于节径数为m的振动位移可表示为

$$w(r,\theta,t)=W(r)\cos m\theta\sin\omega t \tag{5}$$

这样把二维问题化成一维问题处理。

由边界条件(4.a)，(4.b)，(4.c)式看到，各边界条件只与四个状态参量$\{w,\partial w/\partial r,Mr,Pr\}^T$有关。其它边界条件也可用这四个状态参量表示。因此我们可从内力与位移之间关系式（1）和平衡程（3）导出关于四个状态参量的一阶微分程组

$$\begin{aligned}
&\frac{dW(r)}{dr}=\Theta(r)\\
&\frac{d\Theta(r)}{dr}=\frac{\mu m^2}{r^2}W(r)-\frac{\mu}{r}\Theta(r)-\frac{M_r(r)}{D}\\
&\frac{dM_r(r)}{dr}=\frac{m^2D(1-\mu)(3+\mu)}{r^3}W(r)+\frac{D}{r^2}(\mu-1)(2m^2+1+\mu)\Theta(r)\\
&\qquad+\frac{\mu-1}{r}M_r(r)+\frac{P_r(r)}{r}\\
&\frac{dP_r(r)}{dr}=\left[\frac{Dm^2(1-\mu)(2+m^2+\mu m^2)}{r^3}+\frac{m^2}{r}N_\theta-\frac{\mu m^2}{r}N_r\right.\\
&\qquad\left.-\rho hr\omega^2\right]W(r)-\left[\frac{Dm^2(1-\mu)(3+\mu)}{r^2}+(1-\mu)N_r\right.\\
&\qquad\left.+r\frac{dN_r}{dr}\right]\Theta(r)+\left(\frac{\mu m^2}{r}+\frac{rN_r}{D}\right)M_r(r)
\end{aligned}\tag{6}$$

方程组（6）在相应边界条件下用Runge-Kutta法求解。今以一刚性固定在轴上的涡轮盘为例说明计算方位。此时认为内孔边固定，轮缘自由，即

$$W\Big|_{r_0}=\Theta\Big|_{r_0}=0,\qquad M_r\Big|_R=P_r\Big|_R=0 \tag{7}$$

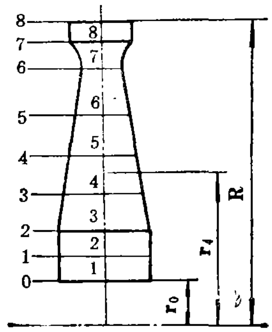

图1　轮盘计算模型

将轮盘分成N个圆环，如图1所示。计算从内孔边开始，由边界条件知，起始点的四个状态参量有两个为零，另两个为未知，设为M_{ro}，P_{ro}。

设试算频率为ω，节径数m。根据线性叠加原理，可分两步积分，轮流使M_{ro}，P_{ro}其中一个为1，另一个为零，用Runge-Kutta法积分方程(6)，计算每站的状态参量，一直算到末站(轮缘)得末站的状态参量，它们是初始状态参量M_{ro}，P_{ro}的线性函数，即

$$\{W\ \Theta\ M_r\ P_r\}^T=\begin{bmatrix}a_{11}&a_{21}&a_{31}&a_{41}\\a_{12}&a_{22}&a_{32}&a_{42}\end{bmatrix}^T\begin{Bmatrix}M_{ro}\\P_{ro}\end{Bmatrix} \tag{8}$$

由轮缘处自由边界条件得

$$\begin{Bmatrix} M_r \\ P_r \end{Bmatrix} = \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \begin{Bmatrix} M_{ro} \\ P_{ro} \end{Bmatrix} = [A] \begin{Bmatrix} M_{ro} \\ P_{ro} \end{Bmatrix} \tag{9}$$

方程（9）要有非零解，其系数行列式值为零

$$\Delta(\omega) = \det[A] = 0 \tag{10}$$

这是多项式求根的问题，可用区间对分法，牛顿法或Muller（抛物线法）法求根，得自振圆频率ω。再由方程组（9）求出M_{ro}，P_{ro}的比例解，然后回代求得每站的四个状态参量，得振型

$$w(r,\theta) = W(r)\cos m\theta \tag{11}$$

其它边界条件的计算步骤一样，只是初始状态参量和频率行列式要根据边界条件而定。

二、基于厚板理论的轮盘振动数值计算

1951年Mindlin和Derisicwicz提出了计算厚板的理论，1954年他们应用此理论计算厚圆板的振动[2]。在近期的轮盘振动有限元分析中普遍采用了这一理论。

厚板理论考虑了剪切和转动惯性的影响。板中面法线变形后绕r，θ轴的转角ψ_r，ψ_θ在数值上不再是$\partial w/\partial r$和$\partial w/r\partial\theta$。位移可表示成

$$u = -z\psi_r(r,\theta,t), \qquad v = -z\psi_\theta(r,\theta,t), \qquad x = w(r,\theta,t) \tag{12}$$

内力和内力矩与变形之间的关系[5]为

$$\begin{aligned} M_r &= -D[\partial\psi_r/\partial r + \mu(\partial\psi_\theta/r\partial\theta + \psi_r/r)] \\ M_\theta &= -D[\partial\psi_\theta/r\partial\theta + \psi_r/r + \mu\partial\psi_r/\partial r] \\ M_{r\theta} &= -\frac{1}{2}D(1-\mu)[\partial\psi_r/r\partial\theta - \psi_\theta/r + \partial\psi_\theta/\partial r] \\ Q_r &= C(\partial w/\partial r - \psi_r) \\ Q_\theta &= C(\partial w/r\partial\theta - \psi_\theta) \end{aligned} \tag{13}$$

式中，$C = (\pi^2/24)Eh/(1+\mu)$，为圆板的剪切刚度

若考虑转动惯性的影响，只要在薄板的平衡方程（3）中第一，第二式的左边分别加上单位长度的惯性力矩$\rho J\partial^2\psi_r/\partial t^2$和$\rho J\partial^2\psi_\theta/\partial t^2$，第三式不变，即为厚板的平衡方程。其中：$J = h^3/12$。

此时厚圆板的边界条件为

$$\begin{aligned} &\text{固定边：} W = 0;\ \psi_r = 0;\ \psi_\theta = 0 \\ &\text{简支边：} W = 0;\ \psi_\theta = 0;\ M_r = 0 \\ &\text{自由边：} M_r = 0;\ M_{r\theta} = 0;\ Q_r = 0 \end{aligned} \tag{14}$$

同样位移的θ函数可用富里叶级数表达，对于节径数为m的振动位移可表示为

$$\begin{aligned} W(r,\theta,t) &= W(r)\cos m\theta \sin\omega t \\ \Psi_r(r,\theta,t) &= \Psi_r(r)\cos m\theta \sin\omega t \\ \Psi_\theta(r,\theta,t) &= \Psi_\theta(r)\sin m\theta \sin\omega t \end{aligned} \tag{15}$$

边界条件与六个状态参量$\{W \ \ \Psi_r \ \ \Psi_\theta \ \ M_r \ \ M_{r\theta} \ \ Q_r\}^T$有关。因此可从(13)，(15)式及厚板平衡方程得到六个关于状态参量的一阶微分方程

$$\frac{dW}{dr}=\frac{Q_r}{C}+\Psi_r \qquad \frac{d\Psi_r}{dr}=-\frac{\mu}{r}-\frac{\mu m}{r}\Psi_\theta-\frac{M_r}{D}$$

$$\frac{d\Psi_\theta}{dr}=\frac{m}{r}\Psi_r+\frac{\Psi_\theta}{r}-\frac{2M_{r\theta}}{D(1-\mu)}$$

$$\frac{dM_r}{dr}=-\left[\frac{D(1-\mu)^2}{r^2}-\rho J\omega^2\right]\Psi_r-\frac{D(1-\mu^2)m}{r^2}\Psi_\theta-\frac{1-\mu}{r}M_r-\frac{m}{r}M_{r\theta}+Q_r \tag{16}$$

$$\frac{dM_{r\theta}}{dr}=-\frac{cm}{r}W-\frac{D(1-\mu^2)m}{r^2}\Psi_r-\left[\frac{D(1-\mu^2)m^2}{r^2}+C-\rho J\omega^2\right]\Psi_\theta+\frac{\mu m}{r}M_r-\frac{2M_{r\theta}}{r}$$

$$\frac{dQ_r}{dr}=\frac{C}{C+N_r}\left\{\left[\frac{m^2(C+N_\theta)}{r^2}-\rho h\omega^2\right]W-\left[\frac{(1-\mu)N_r}{r}+\frac{dN_r}{dr}\right]\Psi_r+\frac{m(C+\mu N_r)}{r}\Psi_\theta+\frac{N_r}{D}M_r-\left[\frac{C+N_r}{Cr}+\frac{1}{C}\frac{dN_r}{dr}\right]Q_r\right\}$$

表1 周边固定圆板固有频率（HZ）

模态 节径	模态 节圆	解析解	有限元解	本文薄板解
0	0	123.3	122.5	124.7
	1	480.5	470.0	492.8
	2	1076.2	1036.5	1120.0
1	0	256.8	253.4	264.0
	1	734.6	713.0	767.0
	2	1450.3	1389.6	1544.1
2	0	421.3	412.7	423.2
	1	1021.6	982.7	1042.9
	2	1857.8	1768.3	1954.4
3	0	616.5	598.3	620.2
	1	1340.7	1278.1	1360.0
	2	2298.5	2170.6	2369.8

表2 厚板固有频率（HZ）

	板尺寸			计算频率		试验
	r_0(cm)	R(cm)	h(cm)	有限元	本方法	频率
实心自由板	0	23.81	8.89	1831	1864.6	1880
空心自由板	21.11	23.81	8.89	632	632.5	640

注：有限元计算频率和试验频率取自文献〔3〕

表3 涡轮盘固有频率（HZ）

模态 节径	模态 节圆	有限元素法[4] 5单元	有限元素法[4] 10单元	本文厚板解	试验值[4]
0	1	1037	1026	1017	992～1005
	2	3553	3575	3576	
	3	7051	6959	7054	
	4	12644	11870	12071	
1	1	1589	1560	1583	1488～1499
	2	4441	4461	4458	
	3	7505	7410	7500	
	4	12509	11830	11988	
2	0	679	678	671	657～666
	1	2673	2670	2700	
	2	5930	5897	5953	
	3	9583	9472	9500	
3	0	1118	1110	1114	999～1008
	1	4070	4103	4105	
	2	7643	7571	7650	
	3	12211	11798	11886	
4	0	1669	1660	1675	1379～1393
	1	5166	5189	5214	
	2	9500	9398	9471	
	3	14458	13671	13828	

在相应的边界条件用 Runge-Kutta 法积分一阶微分方程组(16)，利用轮缘边界条件得频率方程。具体计算方法同前，不同的是状态参量和微分方程有六个。

三、算 例

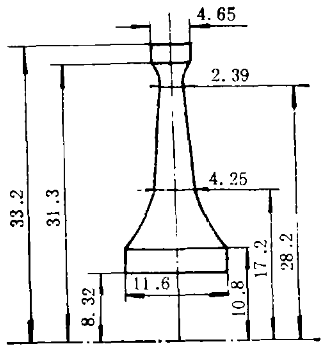

图 2 涡轮盘 ($E=2.2\times10^6$kg/cm^2，$\mu=0.28$，$\rho=0.8388$kg·s^2/cm^4，盘尺寸以cm为单位)

用薄板理论对一个半径为R = 100cm，厚度h = 5 cm，周边固定的实心均匀圆板进行了固有频率计算。材料性质：$\rho=0.795\times10^{-5}$kg·s^2/cm^4，$E=2\times10^6$kg/cm^2，$\mu=0.3$。计算结果列于表 1，并与厚板理论有限元（3 节点 9 自由度环形单元）解[6]、薄板理论解析解作了比较。由表 1 可见，结果令人满意。

为考核本文提出的厚板理论数值计算方法，对一实心自由圆厚板和一环形厚板进行了计算。材料特性：$\rho=0.798\times10^{-5}$kg·s^2/cm^4，$E=2.11\times10^6$kg/cm^2，$\mu=0.3$。计算结果列于表 2，并与有限元解[3]和试验值进行比较，结果十分接近。

C·A·Mota Soares和M·Petyt利用 2 节点12个自由度厚环元素计算了如图 2 所示涡轮盘的固有频率[4]。表 3 列出了将盘分成 5 个单元和10个单元时的固有频率计算值，并与试验值比较。用本文提出的厚板理论数值计算方法计算同一涡轮盘的固有频率。计算结果也列于表 3。由表 3 看到，本文计算结果与厚板有限元解也是很吻合的。

四、讨 论

本文分析了变厚度轮盘振动的数值计算方法。该方法不像有限元素法那样需要求比较复杂的大型矩阵特征值，可以在感兴趣的范围内求得轮盘的振动频率，因此编制程序简单。例如，同时可求按薄板理论和厚板理论盘的振动频率程序，用FORTRAN 语言程序不到二百条。所需存贮量少，省机时，可在微机上实现。

计算精度也达到令人满意的程度，与有限元素法相比有相同的精度。而所使用的计算方法是传统的计算方法，易为工程技术人员掌握。

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（责任编辑李其汉）

INVESTIGATION ON THE NON-SYNCHRONOUS RESPONSES OF FLEXIBLE ROTOR——UNCENTRALIZED SQUEEZE FILM DAMPER BEARINGS SYSTEM

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Abstract

The characteristics of the non-synchronous response of a flexible rotor——uncentralized squeez film damper bearings system are investigated theoretically and experimentally. In the result it is found that:

1. In case of unsuitable selection of system parameters, the system responses vary with the increase in speed ratio from harmonic to 1/2 subharmonic, then harmonic response follows again and is succeeded by the sub-harmonic responses of 1/3, 1/4, 1/5, 1/6, …consecutively. In some cases, the procession of the whole non-synchronous orbit is observed. The non-synchronous responses can be suppressed by choosing suitable system parameters.

2. Medium value of bearing parameter B should be chosen, too big or too small value of B may cause non-synchronous and instable responses. $0.15<B<0.4$ are recommended.

Generally speaking, the larger the unbalance parameter U, the easier for the non-synchronous to occur. But too small U may cause too big transmissibility. So that, $0.1<U<0.3$ are suggested.

3. The sub-harmonic components of the non-synchronous responses will decrease with increasing mass ratio and external damping ratio and with decreasing gravity parameter.

CALCULATION OF TURBINE DISK VIBRATION CHARACTERISTICS

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Abstract

The vibration of turbine disk is analysed using thin plate and Mindlin thick plate theory. The displacement function in circumferential direction can be represented by means of Fourier Series, so that the two-dimensional problem becomes one-dimensional.

Based on the governing equations of plate the equivalent one order differential equation sets can be derived. The frequency equation can be obtained using numerical integration method, such as Runge-Kutta method, under certain boundary conditions, Muller method to solve the resonant frequency of disk is also employed.

The calculation results agree better with experimental data than the results of the finite element method. Moreover, the computer program can be significantly simplified and the computer time can be considerably reduced.

RESEARCH ON OVERLOAD TO STOP CRACKING FOR USED COMPRESSOR BLADES

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Abstract

In order to increase the service life of compressor blades, the effects of overload on fatigue crack formation life in the stainless steel compressor blades which have served for 840 hours and have had some damage on its surface have been investigated. The results show that the residual fatigue life of blades increases with the increasing overload ratio m, and when the overload ratio $m \geqslant 1.6$, the residual life of blades is obviously increased. But when $m < 1.48$, the effects of overload on blade life is not pronounced.

TRANSONIC VIBRATION INDUCED BY MISMATCHING BETWEEN ENGINE AND THRUST NOZZLE

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Abstract

The transonic vibration occured in the flight tests of some combat aircraft at H = 3000 – 4000m and Ma = 0.82 – 0.96 is analyzed in this paper. For nozzle pressure ratio of about 2.9 – 4.6, the vibration frequency ranged from 8 to 12Hz. Some measures were taken to discover the vibration source, such as eliminating the clearance of the aircraft structure, blocking air inlet apertures in the tail section of the aircraft, streamline tracing with power and strapping, and model testing in wind tunnel, but all of them failed.

On the basis of extensive investigation and calculations, it was decided to make simulation test of the ejector nozzles with increased secondary flow rate or with the nozzle shroud apertured for third flow. Finally it was found out that the transonic vibration was induced by mismatching of engine with thrust nozzle.